Exercícios de casa resolvidos

Extensivo — Caderno 3 – Matemática II

Aula 11

7. $sen^2\,\alpha + cos^2\,\alpha = 1 \rightarrow a^2 + cos^2\,\alpha = 1 \rightarrow cos^2\,\alpha = 1 - a^2 \rightarrow cos\,\alpha = \sqrt{1 - a^2}$ (α pertence ao 1° quadrante e, portanto, cos α > 0).

$$tg\,\alpha = \frac{sen\,\alpha}{cos\,\alpha} = \frac{a}{\sqrt{1-a^2}}$$

$$tg\,(\pi - \alpha) = -\,tg\,\alpha = -\frac{a}{\sqrt{1-a^2}}$$

Resposta: A

8. $\begin{cases} sen\,x = \dfrac{m-1}{2} \\ cos\,x = \dfrac{\sqrt{m+2}}{2} \end{cases}$

$$sen^2\,x + cos^2\,x = 1 \rightarrow \left(\frac{m-1}{2}\right)^2 + \left(\frac{\sqrt{m+2}}{2}\right)^2 = 1 \rightarrow \frac{m^2 - 2m + 1 + m + 2}{4} = 1 \rightarrow m^2 - m - 1 = 0$$

$$m = \frac{1 \pm \sqrt{5}}{2}$$

Aula 12

9. $sen^4\,\alpha - cos^4\,\alpha = \frac{1}{4} \rightarrow (sen^2\,\alpha)^2 - cos^4\,\alpha = \frac{1}{4} \rightarrow (1 - cos^2\,\alpha) - cos^4\,\alpha = \frac{1}{4} \rightarrow$

$\rightarrow 1 - 2\,cos^2\,\alpha + cos^4\,\alpha - cos^4\,\alpha = \frac{1}{4} \rightarrow cos^2\,\alpha = \frac{3}{8} \rightarrow cos\,\alpha = \sqrt{\frac{3}{8}}$ (α pertence ao 1° quadrante e, portanto, cos α > 0 e sen α > 0).

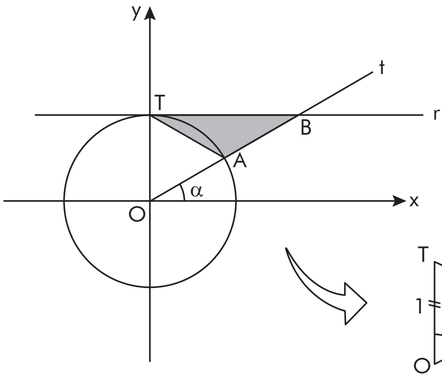
$$sen^2\,\alpha + cos^2\,\alpha = 1 \rightarrow sen^2\,\alpha + \sqrt{\frac{3}{8}}^2 = 1 \rightarrow sen^2\,\alpha = \frac{5}{8} \rightarrow sen\,\alpha = \sqrt{\frac{5}{8}}$$

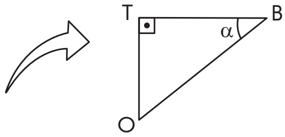
$$tg\,\alpha = \frac{sen\,\alpha}{cos\,\alpha} = \frac{\sqrt{\frac{5}{8}}}{\sqrt{\frac{3}{8}}} = \sqrt{\frac{5}{3}}$$

Resposta: B

Exercícios de casa resolvidos

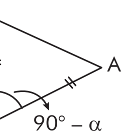
10.





$$\to \operatorname{tg}\alpha = \frac{TO}{TB} \to \operatorname{tg}\alpha = \frac{1}{TB} \to TB = \operatorname{cotg}\alpha$$

$$S_{OTB} = \frac{TB \cdot TO}{2} = \frac{\operatorname{cotg}\alpha}{2} \cdot 1 = \frac{\operatorname{cotg}\alpha}{2}$$



$$\to S_{OTA} = \frac{1}{2} \cdot 1 \cdot 1 \cdot \operatorname{sen}(90^o - \alpha) = \frac{1}{2}\cos\alpha$$

$$S_{TAB} = S_{OTB} - S_{OTA} = \frac{\operatorname{cotg}\alpha}{2} - \frac{\cos\alpha}{2} = \frac{1}{2}\left(\frac{\cos\alpha}{\operatorname{sen}\alpha} - \cos\alpha\right) = \frac{1}{2}\left(\frac{\cos\alpha - \operatorname{sen}\alpha\cos\alpha}{\operatorname{sen}\alpha}\right) =$$

$$= \frac{1}{2} \cdot \frac{\cos\alpha(1 - \operatorname{sen}\alpha)}{\operatorname{sen}\alpha} = \frac{\operatorname{cotg}\alpha(1 - \operatorname{sen}\alpha)}{2}$$

Resposta: D

Aula 13

7. I. $x + y = \frac{\pi}{2} \to y = \frac{\pi}{2} - x \to \operatorname{sen} y = \operatorname{sen}\left(\frac{\pi}{2} - x\right) \to \operatorname{sen} y = \operatorname{sen}\frac{\pi}{2}\cos x - \operatorname{sen} x \cancelto{0}{\cos\frac{\pi}{2}} \to$

$\boxed{\operatorname{sen} y = \cos x}$

$\cos y = \cos\left(\frac{\pi}{2} - x\right) \to \cos y = \cancelto{0}{\cos\frac{\pi}{2}} \cos x + \operatorname{sen}\frac{\pi}{2}\operatorname{sen} x \to$

$\boxed{\cos y = \operatorname{sen} x}$

II. $\operatorname{sen}(y - x) = \frac{1}{3} \to \operatorname{sen} y \cos x - \operatorname{sen} x \cos y = \frac{1}{3}$

Substituindo o que encontramos em I na equação obtida em II, temos:

$$\cos x \cos x - \operatorname{sen} x \operatorname{sen} x = \frac{1}{3} \to \cos^2 x - \operatorname{sen}^2 x = \frac{1}{3} \to \cos^2 x - (1 - \cos^2 x) = \frac{1}{3} \to$$

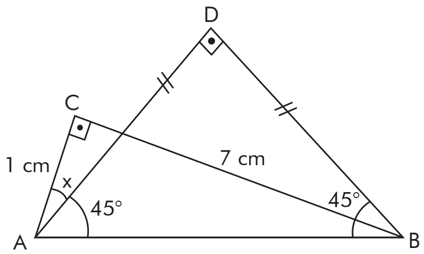
$$\to 2\cos^2 x - 1 = \frac{1}{3} \to \cos^2 x = \frac{4}{6} \to \operatorname{sen}^2 x = \frac{2}{6} \to \operatorname{tg}^2 x = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{1}{2}.$$

Como sen x = cos y e cos x = sen y, temos $\operatorname{sen}^2 x = \cos^2 y$ e $\cos^2 x = \operatorname{sen}^2 y$ e assim $\operatorname{tg}^2 y = \frac{\frac{4}{6}}{\frac{2}{6}} = 2$

$$\operatorname{tg}^2 y - \operatorname{tg}^2 x = 2 - \frac{1}{2} = \frac{3}{2}$$

Resposta: A

9.



$AB^2 = 1^2 + 7^2 \rightarrow AB = \sqrt{50}$

Supondo $C\hat{B}A = \beta$, temos $\text{sen}\,\beta = \dfrac{1}{\sqrt{50}}$ e $\cos\beta = \dfrac{7}{\sqrt{50}}$

Soma dos ângulos internos do triângulo ABC:
$x + 45° + 90° + \beta = 180° \rightarrow x + \beta = 45° \rightarrow x = 45° - \beta \rightarrow$

$\rightarrow \text{sen}\,x = \text{sen}\,(45° - \beta) = \text{sen}\,45° \cos\beta - \text{sen}\,\beta \cos 45° =$

$$= \frac{\sqrt{2}}{2} \cdot \frac{7}{\sqrt{50}} - \frac{1}{\sqrt{50}} \cdot \frac{\sqrt{2}}{2} = \frac{7\sqrt{2} - \sqrt{2}}{2\sqrt{50}} = \frac{6\sqrt{2}}{2\sqrt{50}} = \frac{3\sqrt{2}}{\sqrt{50}} = \frac{3}{5}$$

Resposta: C

Aula 14

3. $\text{tg}\,(x+y) \cdot \text{tg}\,(x-y) = \left(\dfrac{\text{tg}\,x + \text{tg}\,y}{1 - \text{tg}\,x\,\text{tg}\,y}\right)\left(\dfrac{\text{tg}\,x - \text{tg}\,y}{1 + \text{tg}\,x\,\text{tg}\,y}\right) = \dfrac{\text{tg}^2\,x - \text{tg}^2\,y}{1 - \text{tg}^2\,x\,\text{tg}^2\,y} =$

$$= \frac{\frac{9}{16} - \frac{84}{16}}{1 - \frac{9}{16} \cdot \frac{84}{16}} = \frac{-\frac{75}{16}}{-\frac{500}{256}} = \frac{75}{16} \cdot \frac{256}{500} = \frac{12}{5} = 2{,}4$$

Resposta: C

4. I. $\alpha + \beta = \dfrac{\pi}{4} \rightarrow \text{tg}\,(\alpha + \beta) = \text{tg}\,\dfrac{\pi}{4} \rightarrow \dfrac{\text{tg}\,\alpha + \text{tg}\,\beta}{1 - \text{tg}\,\alpha\,\text{tg}\,\beta} = 1 \rightarrow \text{tg}\,\alpha + \text{tg}\,\beta = 1 - \text{tg}\,\alpha\,\text{tg}\,\beta$

II. $(1 + \text{tg}\,\alpha)(1 + \text{tg}\,\beta) = 1 + \text{tg}\,\alpha + \text{tg}\,\beta + \text{tg}\,\alpha \cdot \text{tg}\,\beta$

Substituindo **I** em **II**, temos: $1 + (1 - \text{tg}\,\alpha\,\text{tg}\,\beta) + \text{tg}\,\alpha\,\text{tg}\,\beta = 2$

Resposta: B

Aula 15

5. $\cos x \cos 3x - \text{sen}\,x\,\text{sen}\,3x = \cos(x + 3x) = \cos 4x = \cos 2(2x) = \cos^2(2x) - \text{sen}^2(2x)$

Resposta: A

6. $(\text{sen}\,x + \cos x)^2 = \left(\dfrac{\sqrt{5}}{2}\right)^2 \rightarrow \text{sen}^2\,x + \cos^2 x + 2\,\text{sen}\,x \cos x = \dfrac{5}{4} \rightarrow 1 + 2\,\text{sen}\,x\cos x = \dfrac{5}{4} \rightarrow$

$\rightarrow 2\,\text{sen}\,x \cos x = \dfrac{1}{4} \rightarrow \text{sen}\,x \cos x = \dfrac{1}{8}$

Resposta: A

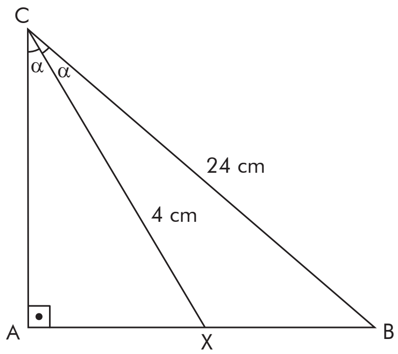
Aula 16

6. $\text{tg}\left(\frac{\pi}{4}+A\right)-\text{tg}\left(\frac{\pi}{4}-A\right)=\dfrac{\text{tg}\frac{\pi}{4}+\text{tg}\,A}{1-\text{tg}\frac{\pi}{4}\text{tg}\,A}-\left(\dfrac{\text{tg}\frac{\pi}{4}-\text{tg}\,A}{1+\text{tg}\frac{\pi}{4}\,\text{tg}\,A}\right)=\dfrac{1+\text{tg}\,A}{1-\text{tg}\,A}-\left(\dfrac{1-\text{tg}\,A}{1+\text{tg}\,A}\right)=$

$=\dfrac{(1+\text{tg}\,A)^2-(1-\text{tg}\,A)^2}{(1-\text{tg}\,A)(1+\text{tg}\,A)}=\dfrac{1+2\,\text{tg}\,A+\text{tg}^2\,A-1+2\,\text{tg}\,A-\text{tg}^2\,A}{1-\text{tg}^2\,A}=\dfrac{2\,\text{tg}\,A}{1-\text{tg}^2\,A}=2(\text{tg}\,2A)=2(5)=10$

Resposta: E

8.



I. $\cos\alpha=\dfrac{AC}{4}$

II. $\cos 2\alpha=\dfrac{AC}{24}\rightarrow 2\cos^2\alpha-1=\dfrac{AC}{24}$

Substituindo **I** em **II**, temos:

$2\left(\dfrac{AC}{4}\right)^2-1=\dfrac{AC}{24}\rightarrow 3\,AC^2-AC-24=0\rightarrow AC=3\text{ cm}$